Quantum mechanics. Department of Physics, 6th semester.

Lesson №3. Mathematical tools of quantum mechanics: Operator function (continuation). Matrix-operators in space E_n. Properties of Pauli matrices.

1. Check hometask.

Tasks 1-2. Calculate eigenfunctions and eigenvalues of operators

$$-i\frac{d}{d\varphi}, \quad -\frac{d^2}{d\varphi^2}, \quad 0 \le \varphi \le 2\pi.$$

2. Operator function.

Task 3. Prove formula: if $\left[\hat{A},\hat{B}\right] = i\alpha$, then $e^{\hat{A}+\hat{B}} = e^{\hat{A}}e^{\hat{B}}e^{-i\alpha/2}$. (ЕК Hl. 1 № 3)

3. Space E_n. Matrix-operators.

3.1. Unit matrix $\delta_{i,k}$

3.2. Hermitian conjugated matrix $(L^\dagger)_{ik} = L^*_{ki}$.

3.3. Hermitian matrix $L_{ik} = L^*_{ki}$

3.4. Unitary matrix $\sum_{i=1}^{n} U^*_{ik}U_{im} = \delta_{km}, \quad \sum_{i=1}^{n} U^*_{ki}U_{mi} = \delta_{km}, \quad |\,DetU_{ik}\,| = 1.$

4. Pauli matrices in space E_2.

$\hat{\vec{\sigma}} = \left(\hat{\sigma}_x,\hat{\sigma}_y,\hat{\sigma}_z\right)$; $\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

Task 4. Prove Hermitean character and unitary property of matrices $\hat{\sigma}_j$. Calculate $\hat{\sigma}_j^2$

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Task 5. Find commutators $\left[\hat{\sigma}_j,\hat{\sigma}_k\right]$ and anti-commutators $\left[\hat{\sigma}_j,\hat{\sigma}_k\right]_+$

Task 6. Find eigenfunctions and eigenvalues $\hat{\sigma}_j$.

Task 7. Find an explicit form of operators $e^{i\varphi\hat{\sigma}_j}$. What is the matrix meaning $e^{i\varphi\hat{\sigma}_y}$?

5. **Quiz** (~ 20 minutes). Two-parts test: 1st part - **5 points**, 2nd part - **10 points**, ut to **15 points** total.

Hometask: *ЕК Hl.1 № 11; НКК № 1.34(v,h)*; for matrices $\sigma_\pm = \hat{\sigma}_x \pm i\hat{\sigma}_y, \hat{\sigma}_z$ calculate $\left[\hat{\sigma}_z,\hat{\sigma}_\pm\right], \quad \hat{\sigma}_\pm^2, \quad \left[\sigma_+,\sigma_-\right], \quad \left[\sigma_+,\sigma_-\right]_+$

1. Find eigenfunctions and eigenvalues of Hermitian operator in space E_2

$$\hat{L} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad (EK\ Hl.1\ №\ 11)$$

2. Find eigenfunctions and eigenvalues of non-Hermitian operators

$$\hat{a} = \begin{pmatrix} 1 & i \\ 0 & 0 \end{pmatrix}, \quad \hat{b} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \ (НКК\ №\ 1.34\ (в,г))$$

3. For matrices $\sigma_{\pm} = \hat{\sigma}_x \pm i\hat{\sigma}_y, \hat{\sigma}_z$

calculate the following commutators $\left[\hat{\sigma}_z, \hat{\sigma}_{\pm}\right], \quad \hat{\sigma}_{\pm}^2, \quad \left[\sigma_+, \sigma_-\right], \quad \left[\sigma_+, \sigma_-\right]_+$

EK – Elyutin P.V., Krivchenko V.D. Quantum mechanics 1976
НКК- Halitskii E.M., Karnakov B.M., Kohan V.I. Problems in Quantum Physics, 1981
Hr. - Hrechko, Suhakov, Tomasevich, Fedorchenko Collection of theoretical physics problems, 1984